Modern Portfolio Theory – Asset Allocation with Matlab

I. Introduction:
Allocating wealth among risky and risk-free assets is one of the main concerns of financial theory, and risk-return trade-offs appear to be a momentum behind any investment decision. Modern portfolio theory arises as a fundamental building block in constructing and optimizing efficient portfolio. Its focus on mean and variance of portfolio returns has provided a great insight into how one will achieve optimal asset allocation that maintains same expected return with minimized risk or maximizes return given a risk preference.

This paper will first discuss the mean-variance analysis and theoretical framework of portfolio optimization including: efficient frontier, capital market line (CML), security market line (SML), and capital asset pricing model (CAPM). Then it will introduce market neutral long-short equity strategy and 130/30 strategy often used in hedge funds. Finally, an example will be provided using data from technology and financial sectors with illustrations and application from Matlab.

II. Efficient market hypothesis and random walk hypothesis:

a. Efficient market hypothesis:
Efficient market hypothesis (EMH) is probably one of the most well-known financial concepts and stands as major assumption in many financial theories. It assumes that financial markets are self-reflected and stock prices therefore reflect all information available. Consequently, one cannot consistently profit by beating the market because in competitive market, this might just be a simple luck. In addition, transaction cost might dry up all the profit as well.

EMH proposes three hypotheses according to the level of information available:
• Weak form: stock prices cannot be predicted from past prices. In other words, historical stock returns cannot be used as a guide to forecast future returns, and thus investors cannot profit from backward analyzing stock prices. The degree of information involved is only that reflected in past stock prices. Under weak form EMH, technical analysis is not valid because it is not possible to trade based on historical stock prices or recent highs and lows.
• Semi-strong form: stock prices cannot be predicted based on past prices and publicly available information such as earnings release, financial statements, etc. The degree of information extends to greater scale; this form rules out fundamental analysis and active management.
• Strong form: stock prices cannot be predicted based on past prices, public and private information. It is apparently too strict to be true as one will always benefit from insider information in order to trade first before the public can take on the trend.

EMH emphasizes the importance of information in future stock returns. According to EMH, investors cannot beat the market all the time, yet why do they keep on seeking alpha and chasing positive returns through every trade? That explained why EMH assumption might only be relative; moreover, there must be some shortcomings regarding this hypothesis. It is obvious that information is valuable and asymmetric among investors. In other words, taking the example of public information, one might find it a little more difficult to obtain while it is not the case for others. That is, information cost can also be another problem that might affect EMH and eat in the potential profit.

b. Random walk hypothesis:
Provided the usefulness of EMH as fundamental condition in many financial theories, it is helpful to move into a more quantitative language of econometrics. Random walk hypothesis
(RWH) is another concept closely related to EMH as it asserts that stock prices cannot be predicted because it follows random walk process (by its name, generally, it can take any value). To simply illustrate random walk process, take a classical example of a drunk, at time $t=0$, he is staying at point A; at time $t=1$, he can either move to the left or right with equal probability. In statistical perspective, forecasting models based on random walk can be stated as follows:

- **Model 1**: stock returns follow geometric random walk process (prices follow lognormal process) and are statistically independent and identically distributed (therefore, volatility is constant)
- **Model 2**: stock returns are assumed to be statistically independent, yet not identically distributed (volatility is deterministic and depends on current price level)
- **Model 3**: stock returns are uncorrelated yet not independent either; volatility is time-varying (volatility in 1 period might affect that in recent periods)

### III. Modern portfolio theory:

#### a. Mean-variance analysis and efficient frontier:

Mean-variance analysis basically lies on the risk-return trade-offs of a rational investor: high returns are associated with greater volatility. There are two main points necessary to mention when discussing portfolio theory.

- First, it analyzes the characteristics of expected returns and risks of individual assets and the basket of those assets, and looks at the correlations of each asset with one another and their contribution to the basket of assets, which then leads to a resounding result regarding the benefits of diversification. The theory states that, by diversifying portfolio with a wide range of assets, one can achieve the same expected return but with lower risk, or equivalently, given a risk profile, one can obtain higher returns by holding a pool of different securities.
- Second, the theory provides an instruction on how to locate and visualize risk-return profile of any pool of assets on graph by repeating the optimization procedure many times which then generates efficient frontier comprising sets of optimal portfolios.

#### i. Mean-variance optimization framework and model:

To begin with, consider a rational investor who looks for positive returns while preferring less risk in his investment. The degree of risk aversion of each investor will play a significant role in his selecting optimal portfolio later on. The two optimization problems often seen are:

- **Maximize portfolio returns given risk preference:**

  Objective function: $\max \Sigma w' \mu$
  
  s.t: $\Sigma w't = 1$
  
  $w' \Sigma w = \sigma^2$

  $w = (w_1, w_2, ..., w_N) \sim$ weight matrix
  
  $\mu = (\mu_1, \mu_2, ..., \mu_N) \sim$ expected return matrix
  
  $t = (1, 1, ..., 1)$

  In a simpler notation, the problem can be written as follows:

  Objective function: $\max \Sigma w_i R_i$

  s.t: $\Sigma w_i = 1$

  $\Sigma \Sigma w_i w_j \text{cov}(i, j)$

  $w_i$: asset weights

  $R_i$: expected return

  In the above problem, short-selling is allowed as we do not constrain portfolio weight to be greater than or equal to 0.
To illustrate mean-variance analysis, consider a portfolio with two assets. The expected return and standard deviation of this portfolio are:

\[
E(p) = wE_A + (1-w)E_B \\
\sigma(p) = \sqrt{[w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\text{cov}(A,B)]} = \sqrt{[w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\rho(A,B)\sigma_A\sigma_B]} \\
\]

Expected return is therefore a linear combination of individual assets while variance is nonlinear combination of variances of component assets and their covariance/correlation. The more assets in portfolio, the harder it is to estimate correlations, yet correlation is the determinant to portfolio volatility and return. Portfolios whose securities have low correlations with each other tend to display low volatility and vice versa. Followings are three special cases of correlation of two-asset portfolio:

- **\( \rho = -1 \)**: securities are negatively correlated
  \[
  \sigma(p) = \sqrt{[w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\rho(A,B)\sigma_A\sigma_B]} = w\sigma_A - (1-w)\sigma_B
  \]
- **\( \rho = 0 \)**: securities have no correlation
  \[
  \sigma(p) = \sqrt{[w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\rho(A,B)\sigma_A\sigma_B]} = \sqrt{[w^2\sigma_A^2 + (1-w)^2\sigma_B^2]}
  \]
- **\( \rho = 1 \)**: securities are perfectly correlated
  \[
  \sigma(p) = \sqrt{[w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\rho(A,B)\sigma_A\sigma_B]} = w\sigma_A + (1-w)\sigma_B
  \]

Investigating the covariance/correlation structure of portfolio and assuming \( N \) assets in portfolio, variance of portfolio is:

\[
\sigma^2(p) = \sum(1/N)\sigma_{n}^2 + \sum\sum(1/N)(1/N)\sigma_{n,m} \\
= (1/N)\sum(\sigma_{n}^2/N) + [(N-1)/N]\sum\sum[\sigma_{n,m}/(N(N-1))] \\
= (1/N)\sigma_{ave}^2 + ((N-1)/N)\sigma_{ave}^2
\]

Increasing \( N \) indefinitely, we have:

\[
\sigma(p) = \sigma_{ave}^{n,m}
\]

Therefore, as we increase the number of assets in portfolio, portfolio variance will decrease to average covariance of assets; this remaining risk is referred to as systematic risk and cannot be diversified away. Nonsystematic risk disappears as more assets are added to portfolio. This proves the benefit of diversification. Moreover, according to the above result, correlation structure of portfolio plays a significant role and is a key factor to analyzing volatility. For large portfolio comprised of hundreds of securities, one method to analyze correlation is Principle Component Analysis (PCA), which helps reduce the dimension of correlation matrix and narrow it down to a few factors mostly explaining the matrix.

- **Other optimization problems in mean-variance analysis**: in addition to the two problems aforementioned, we can derive the same result and weights by solving the following objective functions:
  - **Sharpe ratio maximization**: Sharpe ratio measures the excess return of security/portfolio over standard deviation
    \[
    \text{Sharpe} = (R_i - R_f)/\sigma_i \\
    \text{Objective function: max } (\Sigma w'\mu - \mu_f)/\sqrt{\Sigma w'\Sigma w} \\
    \text{s.t: } \Sigma w'1 = 1
    \]
    \[
    \text{Objective function: max } (\Sigma w_iR_i - R_f)/\sqrt{\Sigma \Sigma w_iw_j \text{cov}(i, j)}
    \]
**Risk aversion optimization:**
Objective function: \[ \text{max } w'\mu - \lambda w'\Sigma w \]
s.t: \[ \Sigma w_i = 1 \]
\( \lambda \): risk aversion index
Lower \( \lambda \) corresponds to lower risk aversion and exposure volatility, which then results in higher returns.

**ii. Efficient frontier:**
Efficient frontier is a visual representative of all feasible and optimal sets of portfolios generated by mean-variance analysis. For any preferred risk/return, the corresponding point on efficient frontier represents the portfolio, which has max return at specified risk or min volatility at given expected return. By solving one of the above optimization problems repeatedly many times, we can obtain efficient frontier as illustrated below:

![Efficient Frontier](image)

The area below and to the right of efficient frontier is feasible area including all portfolios that can be constructed yet not efficient while portfolios above and to the left are impossible.

**b. Capital market line (CML):**
The concept of CML arises when we include risk-free asset in our risky portfolio. Assuming investor can borrow and lend at risk-free rate. The choice of risk-free asset rate depends on investment horizon of different investors; it can be 1 month, 1 year, etc. CML connects risk-free point on y-axis to efficient frontier and is tangent to efficient frontier. Following is a graph of efficient frontier and CML. The tangent point is called market portfolio because it consists of all risky assets whose weight is:

\[ w_i = (p_i \times n_i) / \sum p_i \times n_i \]
According to the graph, portfolio lying on CML is superior to that on efficient frontier except for tangency point. Therefore, a combination of risk-free and risky assets will result in higher returns for any standard deviation. Part of the line from the intercept to tangency point consists portfolios with risk-free and risky assets while the remaining part represents portfolios with risky assets funded by borrowing at risk-free rate. The more risk averse, the greater expected return obtained along CML. The expected return and standard deviation of portfolio on CML are as follows:

\[
\begin{align*}
E(p) &= (1-w_M)R_f + w_ME(R_M) = R_f + w_M(E(R_M) - R_f) \\
\sigma(p) &= \sqrt{w_f^2\sigma_f^2 + w_M^2\sigma_M^2 + 2w_fw_M\text{cov}(R_f, R_M)} = w_M\sigma_M
\end{align*}
\]

Therefore: 
\[
E(p) = R_f + \left(\frac{\sigma_p}{\sigma_M}\right)(E(R_M) - R_f)
\]

In the expression above, \(\frac{(E(R_M) - R_f)}{\sigma_M}\) is market price of risk because it represents the incremental return for additional risk. This formula is similar to Sharpe ratio except for the expected return is expected market return, not individual security return.

c. Security market line (SML) and CAPM:

i. Security market line (SML):

SML and CML are two concepts that are closely related in mean-variance analysis and CAPM. CML represents portfolios whose expected returns are linear function of expected market return and do not relate to individual asset returns. On the other hand, SML represents risk-return relation of individual assets. In other words, individual securities lie on SML (not on CML except for efficient portfolios) and their expected returns are measured:

\[
E(R_i) = R_f + \beta(E(R_M) - R_f) = R_f + (E(R_M) - R_f)\text{cov}(R_i, R_M)/\text{var}(R_M)
\]

Beta (\(\beta\)) measures the sensitivity of individual security to market movement, i.e, its contribution to market risk that cannot be diversified, and therefore, represents systematic risk of stock. The major difference between CML and SML expected return formula lies on \(\beta\) and \(\sigma\), which are systematic and total risk, respectively. The relation between these risks is illustrated below:

\[
\sigma^2_j = \beta^2_j\sigma^2_M + \sigma^2_{ej}
\]
Additionally, slope of CML is market price of risk \( \frac{E(R_M) - R_f}{\sigma_M} \) while that of SML is \( \frac{E(R_M) - R_f}{\beta} \). Following is a plot of SML:

![SML Plot]

Beta of each individual security is estimated using regression method of excess returns of individual security against excess returns of market portfolio, resulting in characteristic line:

\[
R_i - R_f = \alpha + \beta(R_M - R_f) + \epsilon_i
\]

\( \epsilon_i \): is error term (residuals) of linear regression

The characteristic line from linear regression includes alpha term \( \alpha \), which is stock return in excess of return by accepting extra risk (risk premium). In other words, one has higher return as \( \alpha \) increases and thus, he might not need to take more risk. That is why investors constantly seek stock alpha. According to efficient market hypothesis mentioned previously, expected value of \( \alpha \) is 0 because information is fully reflected in price and stock cannot display extraordinary return.

Incorporating \( \alpha \) in explaining the characteristic line and stock return, we have three cases:

- \( \alpha > 0 \): stock return is little compared to risk taken
- \( \alpha = 0 \): stock return is commensurate to risk taken
- \( \alpha < 0 \): stock return exceeds the risk taken

Three ranges of \( \beta \):

- \( \beta > 1 \): securities have higher risk than market portfolio
- \( \beta = 1 \): securities have same risk as market portfolio
- \( \beta < 1 \): securities have lower risk than market portfolio

That is to say, when market is bullish, one tends to invest in high beta stocks in order to get higher returns for taking extra risk; when market is bearish, low beta stocks are preferred as market is volatile.

**ii. Capital asset pricing model (CAPM):**

CAPM is a one-factor model measuring return of individual stock and used extensively in real life by academia and professionals. As previously discussed in SML section, CAPM formula is:

\[
E(R_i) = R_f + \beta(E(R_M) - R_f)
\]
Following CAPM, investors are rewarded for additional risk they assume. In order for CAPM to work, several assumptions are made:

- Investors are rational and make decisions based on risk-return analysis
- Single-period investment horizon
- No friction,
- Market is competitive
- Investors have same expectations about risk and return of all assets

d. Utility function:

Investors have different preferences on risk and return depending on their risk-averse level. One way to quantify investors’ preference is to use utility function and express it by indifference curve that is often seen in economics. In order to construct an optimal portfolio for each investor according to his preference, it is necessary to match their utility to efficient frontier, and the tangent point is the optimal portfolio being sought.

As illustrated in the above graph, indifference curves represent utility of investor at different level. The more these curves move up and to the left, the greater utility. On any particular indifference curve, utility is the same. The utility function is assumed to be quadratic:

\[ u(x) = x - \frac{b}{2}x^2 \]

Absolute risk aversion: \( r_A(x) = \frac{u''(x)}{u'(x)} \)
Relative risk aversion: \( r_B(x) = -xu''(x)/u'(x) \)

IV. Market neutral and 130/30 strategy:

a. Market neutral strategy:

As implied by its name, market neutral strategy aims at neutralizing various factors of portfolio such as dollar amount, beta, etc. This strategy, similar to other ones, takes advantage of overpriced and underpriced stocks: the investors long underpriced stocks and short overpriced ones such that the net exposure is 0. Dollar neutral strategy is probably one of the
most popular strategies hedge fund managers often use. For example, stock A currently has
10% return and is expected to go up to 15%; stock B has 5% return and is expected to go
down 2% to 3%. An investor wants to long $1 of stock A and short $1 of stock B, i.e. his
portfolio is dollar neutral and gross exposure is $2 dollar while net exposure is 0. The
expected return is therefore 17%, which is higher than long-only strategy of stock A. This is
due to the ability to short sell stock B. Variance of dollar-neutral portfolio:
\[ \text{Var}(xA - yB) = x^2 \text{var}(A) + y^2 \text{var}(B) - 2xy \text{cov}(A, B) \]
Beta neutral strategy is another popular strategy in which one will try to equalize beta of long
and short position, and thus overall portfolio beta is 0 and it is uncorrelated with market
portfolio. Constructing beta-neutral strategy is hard as it requires more statistical knowledge
and application than others.
In general, market neutral strategy aims at neutralizing portfolio’s exposure to systematic risk
and as a result it is only exposed to residuals.

b. **130/30 strategy:**
Most hedge funds use 130/30 long-short equity strategy to increase potential return by
leveraging. Basically, the strategy longs 100% in underpriced stock, shorts 30% in overpriced
stock, and uses proceeds from short position to purchase stock, resulting in 130% long
position. Therefore, it can generate exceptional returns when investors’ forecast and
expectation are favored by market and come out true. However, the downside can be large
due to leverage.

For example, stocks A and B are expected to have 10% and 6% returns, and return on market
portfolio is 8%. Applying 130/30 strategy, we allocate 100% in stock A, short 30% stock B
and take proceed to buy 30% stock A, the return will eventually be 11.2%, beating market by
3.2%.

The ultimate problem is then, how an investor can predict stock price and whether his
prediction is accurate so that these strategies are properly performed for the best result which
matches investor’s utility.

V. **Portfolio optimization – An example in Technology and Financial sectors:**
In this section, we provide an example of the above theoretical materials on real life data. Two
sectors we selected are Technology and Finance. We use Matlab to construct portfolio and solve
optimization problem. Following is the table of stocks used in this example:

<table>
<thead>
<tr>
<th>TECH</th>
<th>ANNUALIZED EXP RETURNS</th>
<th>ANNUALIZED STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOOG</td>
<td>12.18%</td>
<td>34.07%</td>
</tr>
<tr>
<td>AAPL</td>
<td>42.86%</td>
<td>37.34%</td>
</tr>
<tr>
<td>MSFT</td>
<td>7.59%</td>
<td>31.43%</td>
</tr>
<tr>
<td>IBM</td>
<td>17.61%</td>
<td>24.78%</td>
</tr>
<tr>
<td>ORCL</td>
<td>16.78%</td>
<td>33.55%</td>
</tr>
<tr>
<td>CISCO</td>
<td>0.01%</td>
<td>35.30%</td>
</tr>
<tr>
<td>DELL</td>
<td>-4.99%</td>
<td>39.76%</td>
</tr>
<tr>
<td>TXN</td>
<td>8.24%</td>
<td>32.85%</td>
</tr>
<tr>
<td>INTEL</td>
<td>13.41%</td>
<td>33.80%</td>
</tr>
<tr>
<td>Stock</td>
<td>FINANCE</td>
<td>ANNUALIZED EXP RETURNS</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>------------------------</td>
</tr>
<tr>
<td>ADOBE</td>
<td>4.77%</td>
<td>39.41%</td>
</tr>
<tr>
<td>AMZN</td>
<td>43.58%</td>
<td>48.12%</td>
</tr>
<tr>
<td>QCOM</td>
<td>16.78%</td>
<td>35.32%</td>
</tr>
<tr>
<td>HP</td>
<td>-6.42%</td>
<td>34.39%</td>
</tr>
<tr>
<td>BRCM</td>
<td>12.55%</td>
<td>45.33%</td>
</tr>
<tr>
<td>EMC</td>
<td>18.63%</td>
<td>36.11%</td>
</tr>
<tr>
<td>NVDA</td>
<td>7.96%</td>
<td>57.16%</td>
</tr>
<tr>
<td>VRSN</td>
<td>21.63%</td>
<td>40.40%</td>
</tr>
<tr>
<td>ADP</td>
<td>11.13%</td>
<td>24.81%</td>
</tr>
<tr>
<td>AXP</td>
<td>13.27%</td>
<td>49.86%</td>
</tr>
<tr>
<td>BAC</td>
<td>-3.45%</td>
<td>74.69%</td>
</tr>
<tr>
<td>BLK</td>
<td>15.37%</td>
<td>46.67%</td>
</tr>
<tr>
<td>GS</td>
<td>1.65%</td>
<td>50.01%</td>
</tr>
<tr>
<td>JPM</td>
<td>12.94%</td>
<td>55.78%</td>
</tr>
<tr>
<td>MS</td>
<td>1.40%</td>
<td>76.43%</td>
</tr>
<tr>
<td>CITIGROUP</td>
<td>-19.36%</td>
<td>79.17%</td>
</tr>
<tr>
<td>AMTD</td>
<td>11.02%</td>
<td>45.34%</td>
</tr>
<tr>
<td>PNC</td>
<td>15.32%</td>
<td>56.95%</td>
</tr>
<tr>
<td>STT</td>
<td>14.57%</td>
<td>63.73%</td>
</tr>
<tr>
<td>BBT</td>
<td>11.48%</td>
<td>51.44%</td>
</tr>
<tr>
<td>WFC</td>
<td>20.30%</td>
<td>62.02%</td>
</tr>
<tr>
<td>COF</td>
<td>16.94%</td>
<td>65.15%</td>
</tr>
<tr>
<td>AMP</td>
<td>17.64%</td>
<td>56.71%</td>
</tr>
</tbody>
</table>

a. Initial check:
Firstly, we locate each stock on the risk-return graph and check their relative positions with one another. The data we obtained is daily stock prices and returns; so we need to change it to monthly form because Matlab’s Portfolio Optimization Toolbox applies to monthly data.

Assuming 250 days each year, which averages to 20.8 days each month. We multiply this figure with average daily return of each stock to obtain monthly return; to derive monthly standard deviation, multiply daily standard deviation by square root of 20.8. The benchmark used is S&P500. The period we examine is 5 years from 01/01/2007 to 08/16/2012.

The first portfolio we establish is equally weighted portfolio whose components have equal weights. Portfolio weight = \( w_i = \frac{1}{\sum \text{(number of assets)}} = \frac{1}{32} \). Portfolio return = \( w_i R_i \). The following code is entered to generate the plot:

\[
p = \text{Portfolio('AssetList', AssetList);}
\]
\[
p = p.setAssetMoments(AssetMean*20.8, AssetCovar*20.8);
\]
\[
p = p.setInitPort(1/p.NumAssets);
\]
\[
[ersk, eret] = p.estimatePortMoments(p.InitPort);
\]
The risk-return plot shows that equally weighted portfolio has higher return per standard deviation. In this example, we assume risk-free rate is 0. Sharpe ratio of equal weight portfolio is much higher than that of S&P500.

<table>
<thead>
<tr>
<th></th>
<th>RISK</th>
<th>RETURN</th>
<th>SHARPE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EQUAL WEIGHT PORTFOLIO</strong></td>
<td>11.64%</td>
<td>34.99%</td>
<td>3.01</td>
</tr>
<tr>
<td><strong>S&amp;P500</strong></td>
<td>25.41%</td>
<td>3.22%</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Next, we plot the efficient frontier and see how we should allocate our assets. The efficient frontier indicates that our efficient portfolio has way higher return compared to those of other individual assets and seems separate from the crowd of other stocks because it is affected by high returns of Apple and Amazon:
Following is the table of 10 sample portfolios on the efficient frontier obtained from Matlab (for detailed portfolio weights, refer to Excel file attached):

<table>
<thead>
<tr>
<th>ANNUALIZED RETURNS</th>
<th>ANNUALIZED RISK</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.8841%</td>
<td>22.2324%</td>
</tr>
<tr>
<td>17.1835%</td>
<td>22.4402%</td>
</tr>
<tr>
<td>20.4828%</td>
<td>23.0365%</td>
</tr>
<tr>
<td>23.7821%</td>
<td>24.0335%</td>
</tr>
<tr>
<td>27.0815%</td>
<td>25.3774%</td>
</tr>
<tr>
<td>30.3808%</td>
<td>27.0166%</td>
</tr>
<tr>
<td>33.6801%</td>
<td>28.9008%</td>
</tr>
<tr>
<td>36.9795%</td>
<td>31.0314%</td>
</tr>
<tr>
<td>40.2788%</td>
<td>33.4408%</td>
</tr>
<tr>
<td>43.5782%</td>
<td>48.1051%</td>
</tr>
</tbody>
</table>

Follow the code to arrive at the above result:

```matlab
p = p.setDefaultConstraints;
pwgt = p.estimateFrontier(30);
[prsk, pret] = p.estimatePortMoments(pwgt);
clf;
portfolioexamples_plot('Efficient Frontier', ...
Attaching the concept of CML to mean-variance analysis assuming risk-free rate is 0, no short sell, and sum of portfolio weights is 1, we have:

As previously mentioned, the combination of risk-free and risky assets on CML provides greater return than efficient frontier. According to efficient frontier, the range of risk and return:

<table>
<thead>
<tr>
<th></th>
<th>LOWER</th>
<th>UPPER</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ANNUALIZED RETURN</strong></td>
<td>13.86%</td>
<td>43.51%</td>
</tr>
<tr>
<td><strong>ANNUALIZED RISK</strong></td>
<td>22.22%</td>
<td>48.08%</td>
</tr>
</tbody>
</table>

b. **Mean-variance analysis:**
In addition to the initial analysis performed in last section, we can construct portfolio based on our own preference of risk and return. We will solve optimization problems for our target risk and return of 25% and 30%, respectively. Remember to check upper and lower range of return and volatility before placing constraints to see whether they are feasible.

- **30%-return portfolio:**
  
  Objective function: \( \min \sum \sum w_i w_j \text{cov}(i, j) \)
  
  s.t: \( \Sigma w_i = 1 \)
\[ \Sigma w_i R_i = 0.3 \]

- **25%-volatility portfolio:**
  - Objective function: \( \max \Sigma w_i R_i \)
  - s.t. \( \Sigma w_i = 1 \)
  - \( \Sigma \Sigma w_i w_j \text{cov}(i, j) = 0.25 \)

The weights of individual assets of two portfolios mainly distribute among 4 stocks: Apple, IBM, Amazon, and ADP with Apple and IBM being heavily invested.

<table>
<thead>
<tr>
<th>STOCK</th>
<th>WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>30% RETURN</strong></td>
<td></td>
</tr>
<tr>
<td>AAPL</td>
<td>39.96393603</td>
</tr>
<tr>
<td>IBM</td>
<td>37.76231931</td>
</tr>
<tr>
<td>AMZN</td>
<td>11.69261179</td>
</tr>
<tr>
<td>ADP</td>
<td>10.58113286</td>
</tr>
<tr>
<td><strong>25% RISK</strong></td>
<td></td>
</tr>
<tr>
<td>AAPL</td>
<td>31.55254765</td>
</tr>
<tr>
<td>IBM</td>
<td>40.6490173</td>
</tr>
<tr>
<td>AMZN</td>
<td>7.652512751</td>
</tr>
<tr>
<td>ADP</td>
<td>20.1459223</td>
</tr>
</tbody>
</table>

Source code:

```plaintext```
TargetReturn = 0.30;
TargetRisk = 0.25;
awgt = p.estimateFrontierByReturn(TargetReturn/12);
```plaintext```
[arsk, aret] = p.estimatePortMoments(awgt);
bwgt = p.estimateFrontierByRisk(TargetRisk/sqrt(12));
[brsk, bret] = p.estimatePortMoments(bwgt);
clf;
portfolioexamples_plot('Efficient Frontier with Targeted Portfolios', ...
    {'line', prsk, pret}, ...
    {'scatter', [sp500std, ersk], [sp500mean, eret], {'SP500', 'Equal'}}, ...
    {'scatter', arsk, aret, {sprintf('%g%% Return',100*TargetReturn)}}, ...
    {'scatter', brsk, bret, {sprintf('%g%% Risk',100*TargetRisk)}}, ...
    {'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'});
aBlotter = dataset({100*awgt(awgt > 0), 'Weight'}, 'obsnames', ...
    p.AssetList(awgt > 0));
fprintf('Portfolio with %g%% Target Return
', 100*TargetReturn);
disp(aBlotter);
bBlotter = dataset({100*bwgt(bwgt > 0), 'Weight'}, 'obsnames', ...
    p.AssetList(bwgt > 0));
fprintf('Portfolio with %g%% Target Risk
', 100*TargetRisk);
disp(bBlotter);

% Max Sharpe ratio:
% Objective function: max \Sigma w_i R_i / \Sigma \Sigma w_i w_j \text{cov}(i, j)
% s.t: \Sigma w_i = 1

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>73.29</td>
<td></td>
</tr>
<tr>
<td>AMZN</td>
<td>26.71</td>
<td></td>
</tr>
</tbody>
</table>

Since returns on Apple and Amazon are considerably higher than other, they have greater effect in constructing portfolio. As indicated in the graph below, maximized Sharpe ratio portfolio places concentration only on Apple and Amazon.

<table>
<thead>
<tr>
<th>PORTFOLIO TYPE</th>
<th>RISK</th>
<th>RETURN</th>
<th>SHARPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAX SHARPE</td>
<td>35.61%</td>
<td>42.96%</td>
<td>1.21</td>
</tr>
</tbody>
</table>
Incorporating CML:

The following plot confirms that portfolio obtained above has the highest Sharp ratio among others:
Note that Sharpe ratio in the above graph is not yet annualized.

Source code (for full source code, see Appendix):

```matlab
p = p.setInitPort(0);
swgt = p.estimateMaxSharpeRatio;
[srsk, sret] = p.estimatePortMoments(swgt);
clf;
portfolioexamples_plot('Efficient Frontier with Maximum Sharpe Ratio Portfolio', ...,
     {'line', prsk, pret}, ...,
     {'scatter', srsk, sret, {'Sharpe'}}, ...,
     {'scatter', [sp500std, ersk], [sp500mean, eret], {'SP500', 'Equal'}}, ...,
     {'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.'});
```

c. Incorporating additional constraints:
In this section, we incorporate transaction cost and portfolio turnover constraints into the optimization problem. This involves heavy use of mathematical formula and extends to multi-period portfolio optimization problem and beyond the scope of this paper. Therefore, we will roughly go through the intuition and mathematical background needed to understand the optimization problem, and will not dig in the theoretical aspect yet only practical application of them in constructing our portfolio. Later, we will use Matlab to obtain efficient frontiers and portfolio weights.
Turnover refers to the number of times portfolio needs to be rebalanced and is a concept closely related to transaction cost. Assume the cost of buying and selling stocks are $p$ and $q$, respectively. Buying and selling functions are then:

$$b_i = \begin{cases} 
(w_{i,t} - w_{i,t-1})p, & \text{if } w_{i,t} > w_{i,t-1} \\ 0, & \text{if } w_{i,t} \leq w_{i,t-1} 
\end{cases}$$

$$s_i = \begin{cases} 
(w_{i,t} - w_{i,t-1})q, & \text{if } w_{i,t} > w_{i,t-1} \\ 0, & \text{if } w_{i,t} \leq w_{i,t-1} 
\end{cases}$$

The objective function is therefore:

$$\Phi + \sum b_i + \sum s_i$$

$\Phi$: original objective function without transaction cost

Intuitively, transaction cost will reduce returns. According to the following graph, transaction cost roughly reduces returns by 10%. The shape of efficient frontier is basically similar to no-constraint case yet below it.

Incorporating both transaction cost and turnover constraints leads to greater drop in stock returns as illustrated below. Moreover, efficient frontier is significantly shorter and shifts downward.
d. Market neutral portfolio and 130/30 strategy:
   - Dollar neutral portfolio:
Observing the efficient frontier of dollar neutral strategy, apparently, returns can be achieved at higher level. The frontier spreads out widely and on average, return at any level of standard deviation is about 10% higher than in case of original frontier. In addition, the feasible area is also expanded. The portfolio that maximizes Sharpe ratio is created which has 20% and 10% return and volatility, respectively. Following is the long-short position of components:

<table>
<thead>
<tr>
<th>WEIGHT</th>
<th>LONG</th>
<th>SHORT</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOOG</td>
<td>-7.452791799</td>
<td>0</td>
</tr>
<tr>
<td>AAPL</td>
<td>18.84169232</td>
<td>18.84169232</td>
</tr>
<tr>
<td>MSFT</td>
<td>-4.38688442</td>
<td>0</td>
</tr>
<tr>
<td>IBM</td>
<td>8.737363843</td>
<td>8.737363843</td>
</tr>
<tr>
<td>ORCL</td>
<td>6.085404633</td>
<td>6.085404633</td>
</tr>
<tr>
<td>CISCO</td>
<td>-9.140310285</td>
<td>0</td>
</tr>
<tr>
<td>DELL</td>
<td>-9.128847508</td>
<td>0</td>
</tr>
<tr>
<td>TXN</td>
<td>-4.361949281</td>
<td>0</td>
</tr>
<tr>
<td>INTEL</td>
<td>5.243003053</td>
<td>5.243003053</td>
</tr>
<tr>
<td>ADOBE</td>
<td>-4.850507172</td>
<td>0</td>
</tr>
<tr>
<td>AMZN</td>
<td>6.7420998</td>
<td>6.7420998</td>
</tr>
<tr>
<td>QCOM</td>
<td>1.20384155</td>
<td>1.20384155</td>
</tr>
<tr>
<td>HP</td>
<td>-15.18013331</td>
<td>0</td>
</tr>
<tr>
<td>BRCM</td>
<td>-0.64141165</td>
<td>0</td>
</tr>
<tr>
<td>EMC</td>
<td>5.7439573</td>
<td>5.7439573</td>
</tr>
<tr>
<td>NVDA</td>
<td>-0.105397171</td>
<td>0</td>
</tr>
<tr>
<td>VRSN</td>
<td>3.326919795</td>
<td>3.326919795</td>
</tr>
<tr>
<td>AXP</td>
<td>0.208352468</td>
<td>0</td>
</tr>
<tr>
<td>BAC</td>
<td>-3.097815579</td>
<td>0</td>
</tr>
<tr>
<td>BLK</td>
<td>1.027306007</td>
<td>1.027306007</td>
</tr>
<tr>
<td>GS</td>
<td>-7.090638323</td>
<td>0</td>
</tr>
<tr>
<td>JPM</td>
<td>2.793974055</td>
<td>2.793974055</td>
</tr>
<tr>
<td>MS</td>
<td>2.413068617</td>
<td>2.413068617</td>
</tr>
<tr>
<td>CITIGROUP</td>
<td>-5.234853416</td>
<td>0</td>
</tr>
<tr>
<td>AMTD</td>
<td>-1.152608754</td>
<td>0</td>
</tr>
<tr>
<td>PNC</td>
<td>-0.223058841</td>
<td>0</td>
</tr>
<tr>
<td>STT</td>
<td>-1.056732086</td>
<td>0</td>
</tr>
<tr>
<td>BBT</td>
<td>-3.204574094</td>
<td>0</td>
</tr>
<tr>
<td>WFC</td>
<td>8.613110252</td>
<td>8.613110252</td>
</tr>
<tr>
<td>COF</td>
<td>1.594738573</td>
<td>1.594738573</td>
</tr>
<tr>
<td>AMP</td>
<td>3.736070454</td>
<td>3.736070454</td>
</tr>
</tbody>
</table>

- **130/30 strategy:**
  A quick look at two strategies and their graph, we can see that 130/30 strategy generate max-Sharpe portfolio whose Sharpe ratio is higher than that in dollar neutral strategy. Additionally, weight distribution in dollar neutral portfolio strategy covers almost all
individual assets while 130/30 strategy only focuses on outperformed and underperformed stocks. For example, it heavily longs Apple while shorts Citigroup.

<table>
<thead>
<tr>
<th>130/30</th>
<th>WEIGHT</th>
<th>LONG</th>
<th>SHORT</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>59.95165253</td>
<td>59.95165253</td>
<td>0</td>
</tr>
<tr>
<td>IBM</td>
<td>39.41581018</td>
<td>39.41581018</td>
<td>0</td>
</tr>
<tr>
<td>DELL</td>
<td>-3.722395384</td>
<td>0</td>
<td>3.722395384</td>
</tr>
<tr>
<td>AMZN</td>
<td>20.12825008</td>
<td>20.12825008</td>
<td>0</td>
</tr>
<tr>
<td>HP</td>
<td>-2.968440606</td>
<td>0</td>
<td>2.968440606</td>
</tr>
<tr>
<td>NVDA</td>
<td>-0.315355638</td>
<td>0</td>
<td>0.315355638</td>
</tr>
<tr>
<td>ADP</td>
<td>3.144813446</td>
<td>3.144813446</td>
<td>0</td>
</tr>
<tr>
<td>MS</td>
<td>-4.661808141</td>
<td>0</td>
<td>4.661808141</td>
</tr>
<tr>
<td>CITIGROUP</td>
<td>-18.33200023</td>
<td>0</td>
<td>18.33200023</td>
</tr>
<tr>
<td>WFC</td>
<td>7.359473769</td>
<td>7.359473769</td>
<td>0</td>
</tr>
</tbody>
</table>
REFERENCES


